Worthwhile Tasks

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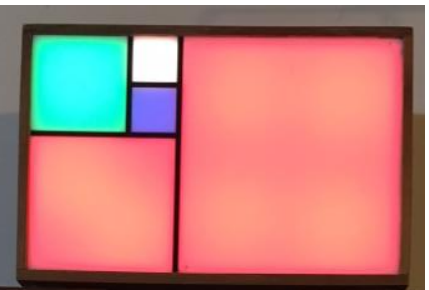
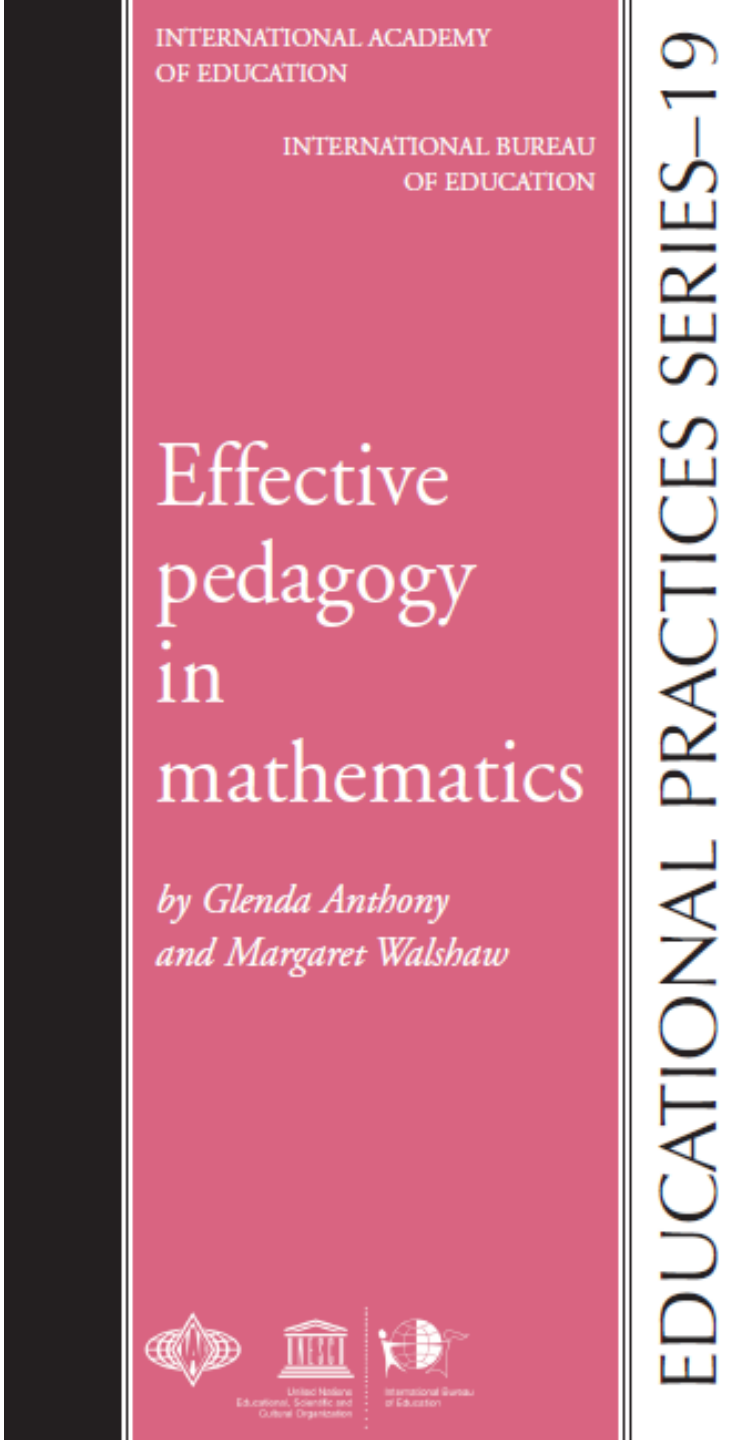




Whakatauki

Ko te herenga waka,
he hakawhitiwhiti korero,
he hakawhitiwhiti whakaaro,
e uu ko te maramatanga

Where canoes are tethered together,
dialogue is exchanged,
thoughts are provoked,
and enlightenment comes forth.

Worthwhile Mathematical Tasks

Effective teachers …. ensure that all students are given tasks that help them improve their understanding

Effective teachers set tasks that require students to make and test conjectures, pose problems, look for patterns, and explore alternative solution paths.

Best Evidence Synthesis: Mathematics – Theme 4…

Worthwhile Mathematical Tasks

Requiring students **to think deeply about mathematical ideas** will ensure that conceptual understanding is developed and that higher order maths thinking is achieved…..

Teachers should **challenge students to explore** concepts (not simply 'get the right answer'), to be able to **generalise** and to develop ideas about the nature of mathematics.

Best Evidence Synthesis: Mathematics – Theme 4…

Worthwhile Mathematical Tasks

This presentation includes examples where pre-planned tasks have provided students with these opportunities to engage with mathematics, along with moments showing how simple tasks can ‘grow a leg’.

We consider the benefits of solving unrehearsed problems and valuing student-generated methods….

It’s not the task…

….. it’s how the task is used

Leave a Gap

- For the community hangi, 356 potatoes had been peeled and there were 233 left to be peeled. How many potatoes will there be altogether?
- Fallon is helping his mum build a path. There were 438 bricks in the pile and they used 169 of them yesterday. How many bricks have they got left for today?

Student-generated methods

Students came up with two strategies:

438 – 169 = (400 -100) + (30 – 60) + (8 – 9)

= 300 – 30 – 1 = 269

or

Jumping up the number line

from 169 to 438:

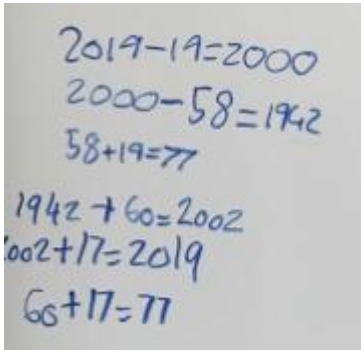
169 + ? = 438

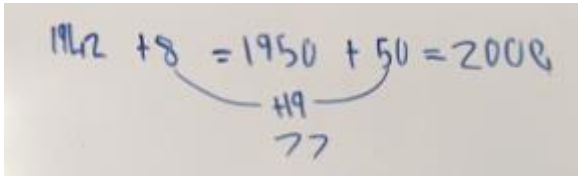
On this day . . .

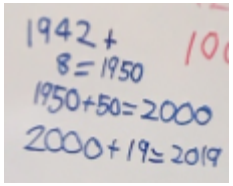
. . . in 1942 about 150 Japanese warplanes attacked the Australian city of Darwin.

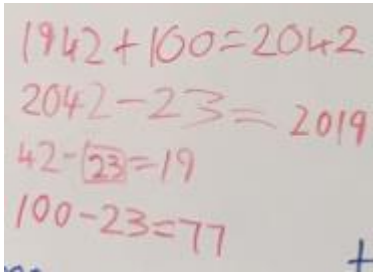
How many years ago was that?

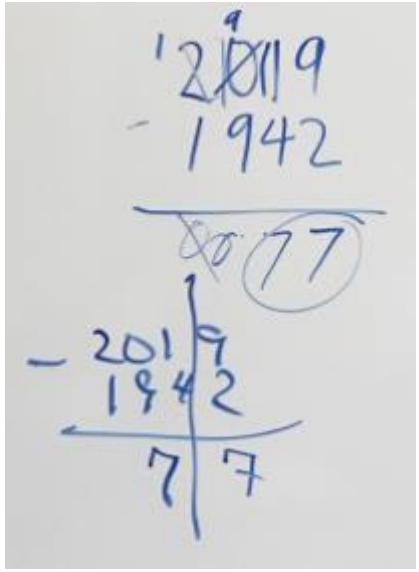
Valuing student-generated solutions

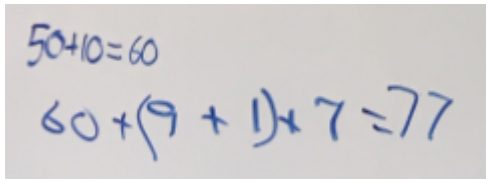


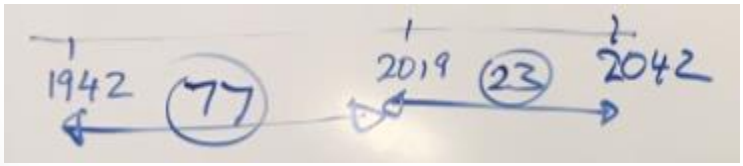


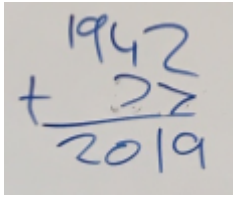












Student-generated methods

What did the students get out of this?

- They learnt that they could make their own sense of the problem
- Their thinking was acknowledged and encouraged.
- They revealed to the teacher an unexpected capability….

They behaved like mathematicians

Fractions – please explain

Explain why $4 \div \frac{2}{3} = 6$

Order these from smallest to largest:

$\frac{8}{9}$ 0.9 $\frac{5}{6}$ 0.88

Explain why $4^{-3} = \frac{1}{64}$

Conceptual Understanding

- comprehension of mathematical concepts, operations, and relations

Prodecural Fluency

- skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

THE STRANDS OF MATHEMATICAL PROFICIENCY." National Research Council. 2001. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: The National Academies Press.

Order these from smallest to largest:

$\frac{8}{9}$ **0.9** $\frac{5}{6}$ **0.88**

What the students did……

- $\frac{5}{6} < \frac{8}{9}$
- $\frac{8}{9} < 0.9$
- $0.88 < \frac{8}{9}$
- $\frac{5}{6} < \frac{88}{100}$

$\frac{5}{6}$ **<** **0.88** **<** $\frac{8}{9}$ **<** **0.9**

Try Cycling . . .

A cycle shop has a total of 36 bicycles and tricycles in stock.

Collectively there are 80 wheels.

How many bikes and how many tricycles are there?

Strategic Competence

- A student with strategic competence can not only come up with several approaches to a nonroutine problem but can also choose flexibly from the methods available to suit the demands presented by the problem and the situation in which it is posed.

Eggsample

There is a number under each egg in this egg carton.

When I remove two eggs I add the numbers together.

It is possible to make every sum from 1 to 10.

What might the six numbers be?

Adaptive Reasoning

- capacity for logical thought, **reflection**, explanation, and justification
- inclusion of metacognition – ie objective reflection around where are we up to in the resolution of this problem / hiccup
- being prepared to step back and assess progress, reconsider the approach, then persist or change tack.

Number of the Day: 356

- Write down a 3 digit number where each of the three digits is different
- Rearrange the digits to make the largest three digit number
- Rearrange the digits to make the smallest three digit number
- Subtract these numbers
- Reverse the answer and add these last two numbers together

653

\- 356

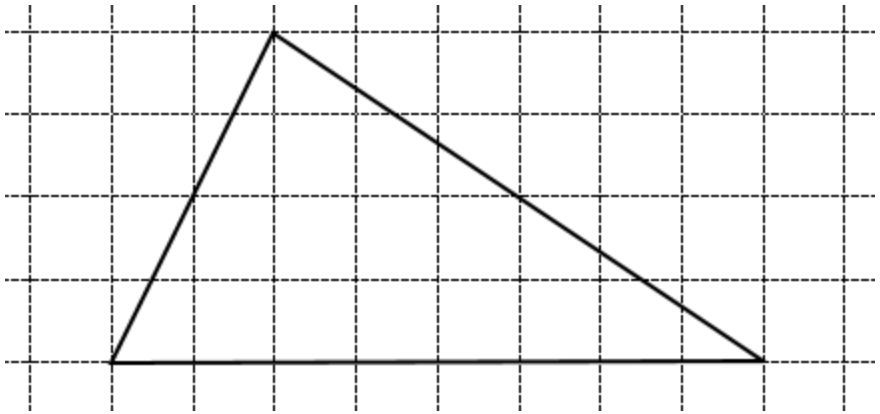
297

\+ 792

1089

Area of a Triangle – students pose problems

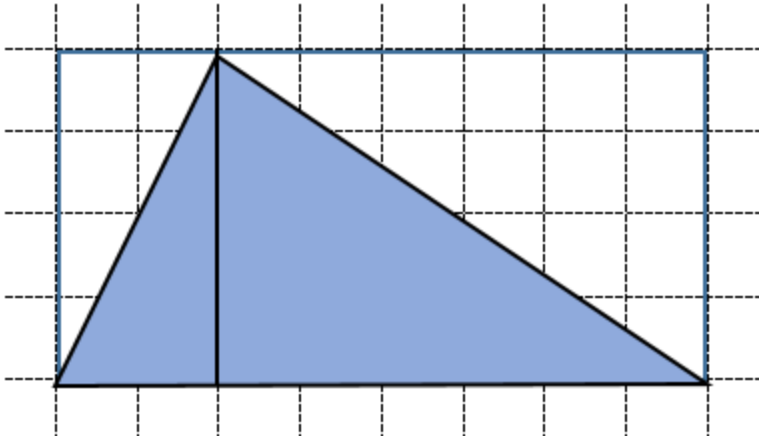
- Draw a triangle:



- How do you calculate the area of the triangle?

Area of a Triangle

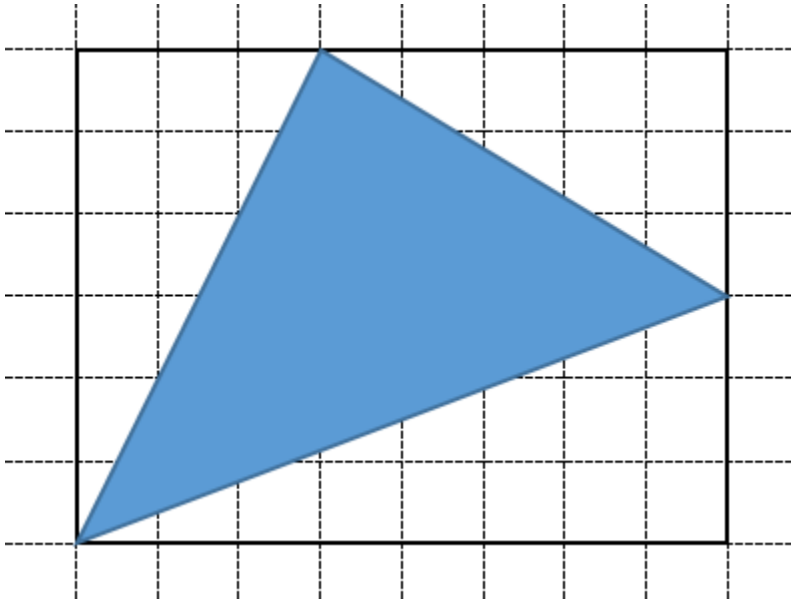
- Surround the triangle with a rectangle:



- Area of triangle = half the area of a rectangle

Area of a Triangle

- Draw any triangle and surround it with a rectangle



- Area of triangle = half the area of a rectangle

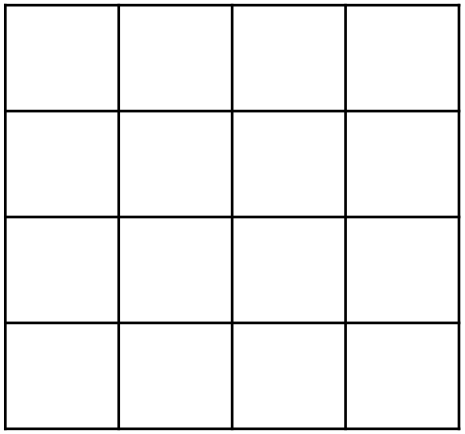
TRUE or FALSE ?

Planting trees

Shane Jones wants 1 billion trees planted in the next 10 years….

Is there a problem?

Counters on a Grid



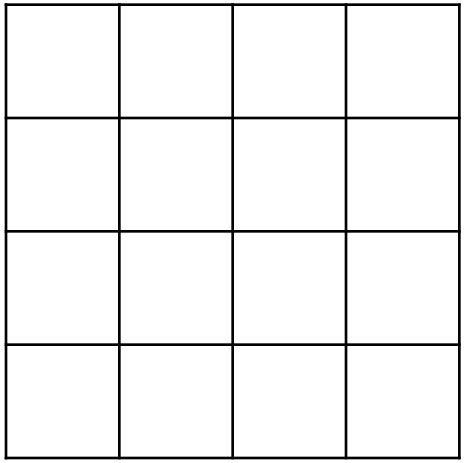
Jon has a 4 x 4 grid of squares on which he is trying to place as many counters as possible.

No more than one counter can be placed on any square and no more than three in any row, column or diagonal.

What is the maximum number of counters he can place in this way?

Four Wolves

A field contains 16 squares.
There are 4 wolves and some sheep in the field,
each on a different square.



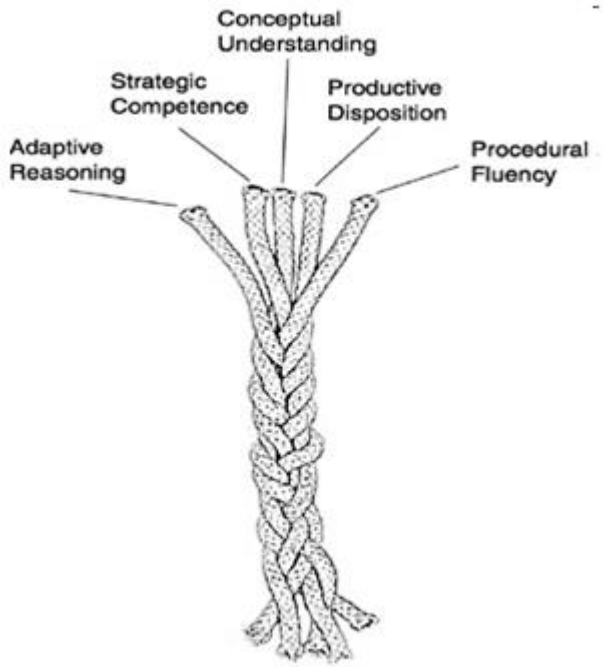
A wolf can eat any sheep that it can see directly in a horizontal, vertical or diagonal line.

How many sheep, each on a different square, can be placed in the field so that they are all safe?

Mathematical Proficiency

Five strands:

- Conceptual Understanding
- Procedural Fluency
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition

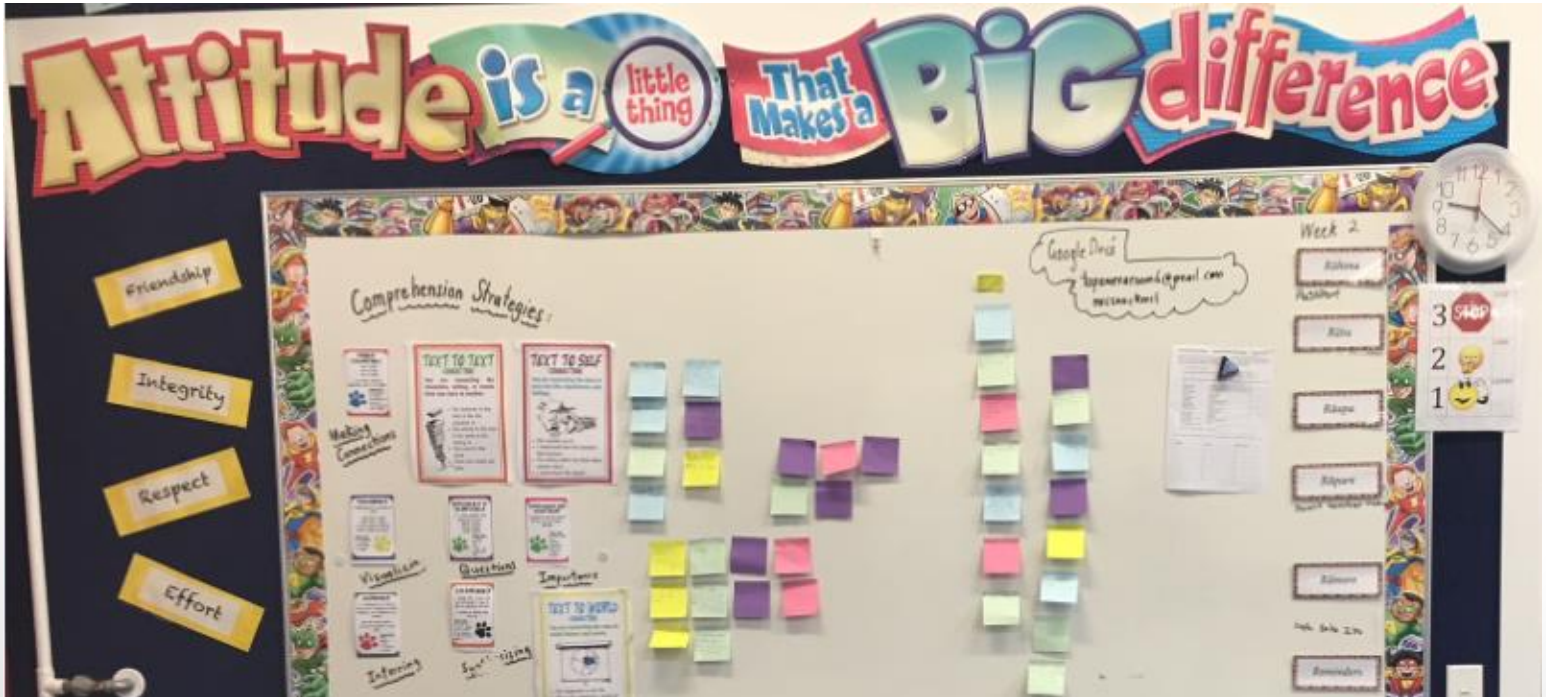


National Research Council 2001

Adding It Up: Helping Students Learn Mathematics

Productive Disposition

- habitual inclination to see mathematics as **sensible**, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy



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Handouts

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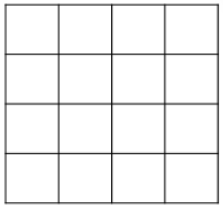

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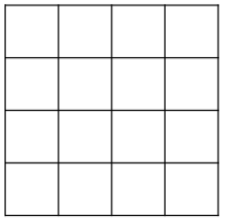

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Planting trees

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Is there a problem?

Kevin, Chris and David decided one day to go fishing …

When they got back they had 13 fish!

How many ways could they have achieved this?